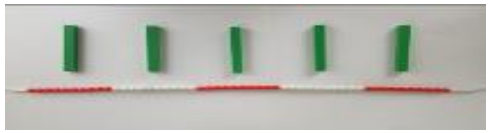
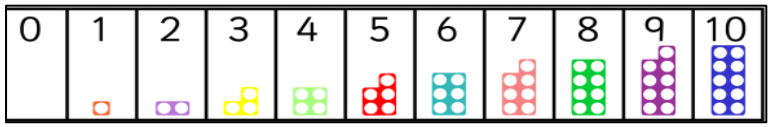
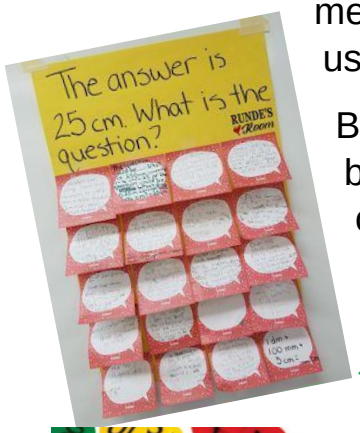
St. Edmund's Progress in Calculation Strategies





Mathematics equips pupils with a uniquely powerful set of tools to understand and change the world. Pupils should make rich connections across mathematical ideas to develop fluency, mathematical reasoning and competence in solving increasingly sophisticated problems. They should also apply their mathematical knowledge to science and other subjects as well as developing their understanding further in the real world.

At St. Edmund's, achieving depth of understanding in maths is supported by: active and collaborative learning without fear of making mistakes; using a Concrete, Pictorial, Abstract approach to develop a deeper understanding of what maths looks like: creating opportunities for children to be purposeful and 'talk maths'; building confidence to be curious about maths through challenging problem solving; allowing the students to be part of their learning journey and make independent choices about the challenges they are provide with: making choices about which mental calculation strategies are the most efficient method to use; showing respect and empathy at all times to other students when using and developing their understanding to become student teachers.



Being secure in maths when leaving primary school is about resilience and being fascinating with learning and not being afraid to make mistakes; our curriculum is designed around building growth mind-sets (Carol Dweck); to ensure students understand the importance of perseverance and taking risks.

There are 12 animals in a farm.

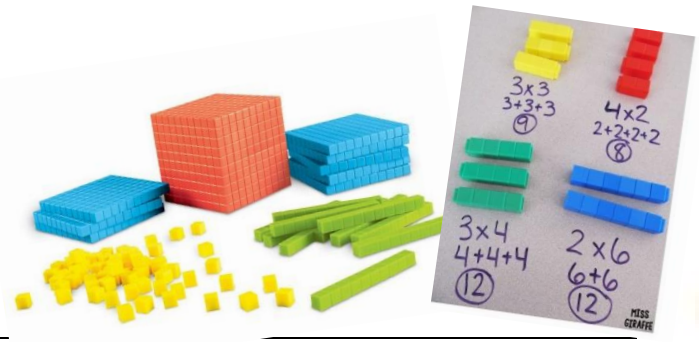
Some are cows

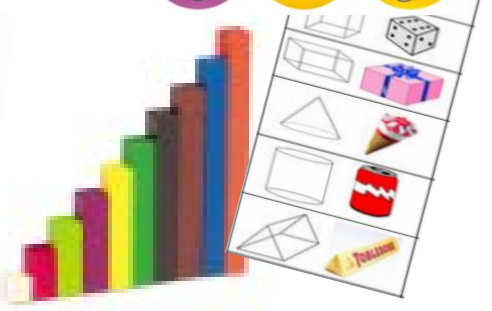
The rest are chickens

There are 28 animal legs in all.

How many cows are there?

How many chickens are there?





Calcula… Guidance: Yearly Overview

	EYFS	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Addition	✓ Counting on in 1s to find one more than a given number ✓ Use concrete objects to understand addition is combining ✓ Regrouping to make 10 using a ten frame	✓ Combining two parts to make a whole: part whole model. ✓ Starting at the bigger number and counting on- using cubes. ✓ Regrouping to make 10 using ten frame.	✓ Adding three single digits. ✓ Use of base 10 to combine two numbers.	✓ Column method- regrouping. ✓ Using place value counters (up to 3 digits).	✓ Column method- regrouping (up to 4 digits)	✓ Column method- regrouping. ✓ Use of place value counters for adding decimals. ✓	✓ Column method- regrouping. ✓ Abstract methods. ✓ Place value counters to be used for adding decimal numbers.
Subtraction	✓ Taking away one by counting back to find one less than a given number ✓ Use 10 frame to take amounts away from.	✓ Taking away ones ✓ Counting back ✓ Find the difference ✓ Part whole model ✓ Make 10 using the ten frame.	✓ Counting back ✓ Find the difference ✓ Part whole model ✓ Make 10 ✓ Use of base 10	✓ Column method with regrouping. ✓ (up to 3 digits using place value counters)	✓ Column method with regrouping. ✓ (up to 4 digits)	✓ Column method with regrouping. ✓ Abstract for whole numbers. ✓ Start with place value counters for decimals- with the same amount of decimal places.	✓ Column method with regrouping. ✓ Abstract methods. ✓ Place value counters for decimals- with different amounts of decimal places.
Multiplication	✓ Recognising and making equal groups ✓ Use cubes, Numicon and other concrete resources to create equal groups and match a Numicon tile to the amount	✓ Recognising and making equal groups. ✓ Doubling ✓ Counting in multiples ✓ Use cubes, Numicon and other objects in the classroom	✓ Arrays- showing commutative multiplication – objects and counters	✓ Arrays ✓ 2d × 1d using base 10	✓ Column multiplication- introduced with place value counters. ✓ (2 and 3 digits multiplied by 1 digit)	✓ Column multiplication ✓ Abstract only but might need a repeat of year 4 first (up to 4 digits numbers multiplied by 1 or 2 digits)	✓ Column multiplication ✓ Abstract methods ✓ (multi-digit up to 4 digits by a 2 digit number)

Division	✓ Sharing object into groups ✓ Use concrete representations and draw around groups of an amount and then match the numicon tile to the amount in the group	✓ Sharing objects into groups ✓ Division as grouping e.g. I have 12 sweets and put them in groups of 3, how many groups? ✓ Use cubes, counters and draw round a given amount then record the expression using symbols and numerals.	✓ Division as grouping ✓ Division within arrays- linking to multiplication ✓ Repeated subtraction	✓ Division with a remainder- using lollipop sticks, times tables facts and repeated subtraction. ✓ 2d divided by 1d using base 10 or place value counters	✓ Division with a remainder ✓ Short division (up to 3 digits by 1 digit- concrete and pictorial)	✓ Short division ✓ (up to 4 digits by a 1-digit number including remainders)	✓ Short division ✓ Long division with place value counters ✓ (up to 4 digits by a 2 digit number) ✓ Children should exchange into the tenths and hundredths column too

Why concrete, pictorial and abstract?

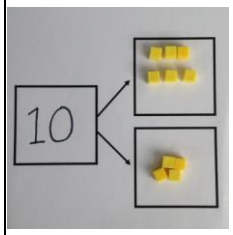
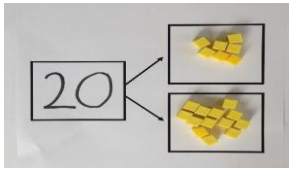
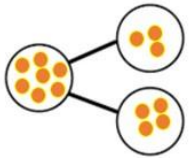
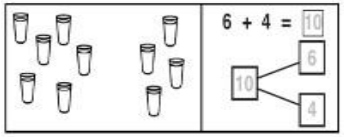
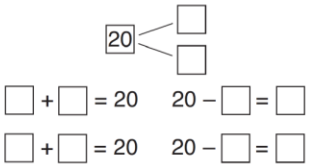
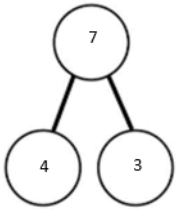
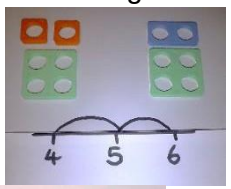
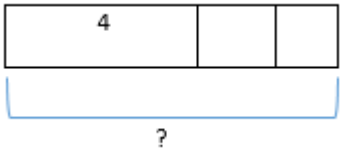
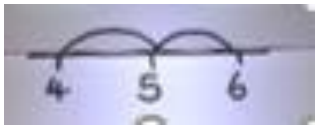
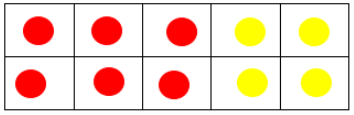
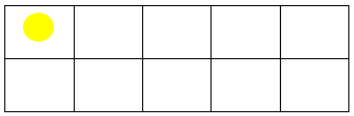
As children become more confident at seeing mathematics and working things out in their heads they become better at problem solving and reasoning as well as calculating or working with shape and measure. Mental arithmetic skills need teaching and practice to develop efficient and effective ways of thinking and organising thoughts and ideas. We need to help children to: carry and manipulate information in their heads; visualise images and to interpret and analyse what they see; select and organise information in a systematic and logical way identifying patterns and applying logical reasoning.

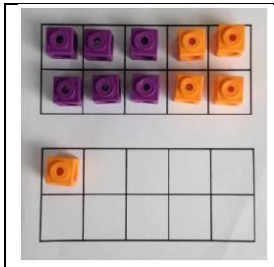
To use visualisation successfully, children need practical experience, along with opportunities to talk about the equipment they are using and the images they are forming in their head. They also need to learn and use the related mathematical language. Visualisation could also involve the children making some notes or jottings to help them. These should not replace the visualisation but provide support when children can no longer hold everything in their minds. Being able to listen to description, interpret the context or task and manipulate the image can be challenging so making jottings of this kind is an important step in the development of their visualisation.

As children develop their fluency, learning may alternate between the concrete, pictorial and abstract areas in order to challenge and stretch student's knowledge and understanding. Across the school addition, subtraction, multiplication and division can be seen in many ways and the students will be encouraged to prove their understanding by using all aspects of the Concrete, Pictorial, Abstract approach throughout their Mathematics lessons as they move through the Key Stages.

Addition-

Key language which should be used: sum, total, parts and wholes, plus, add, altogether, more than, 'is equal to' 'is the same as'

Concrete	Pictorial	Abstract
Combining two parts to make a whole (use other resources too e.g. eggs, shells, teddy bears etc)    	   	4 + 3 = 7 (four is a part, 3 is a part and the whole is seven)  10 = 6 + 4 10 − 6 = 4 10 − 4 = 6 10 = 4 + 6
Counting on using number lines by using cubes, numicon or using a bead string.   	A bar model which encourages the children to count on 	The abstract number line: What is 2 more than 4? What is the sum of 4 and 4? What's the total of 4 and 2? 4 + 2 
Regrouping to make 10 by using ten frames, bead strings and counters/cubes or using numicon: 6 + 5	Children to draw the ten frame and counters/cubes  	Children to develop an understanding of equality e.g 6 + □= 11 and 6 + 5 = 5 + □ 6 + 5 = □+ 4

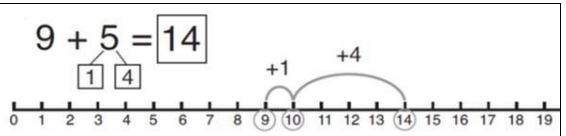


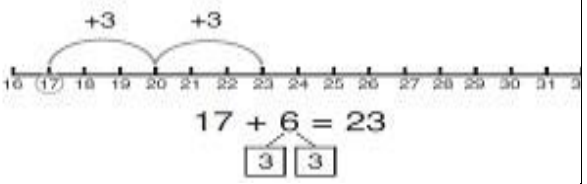
3 + 9 = 12

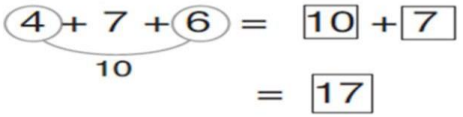
4 + 9 = 13

4 + 7 + 6 = 17

3 + 9 =

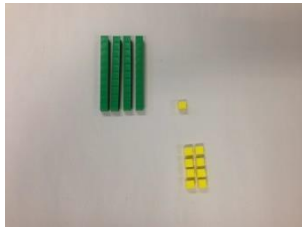




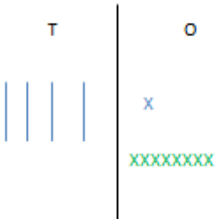


TO + O using base 10. Continue to develop understanding of partitioning and place value

41 + 8



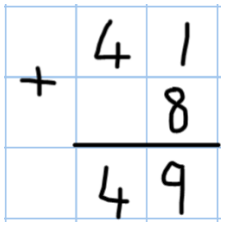
Children to represent the concrete using a particular symbol e.g. lines for tens and dot/crosses for ones.

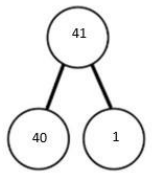


41 + 8

1 + 8 = 9

40 + 9 = 49





<table>
<tr>
<td></td>
<td> 
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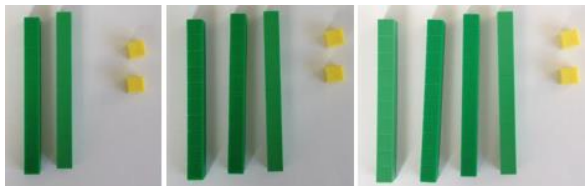
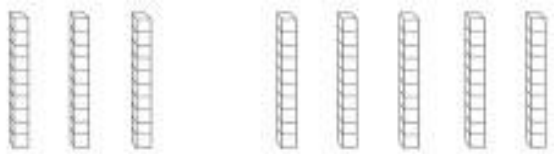
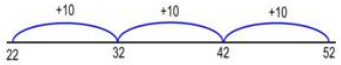
<td>

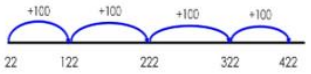
hundreds	tens	ones
3	5	8
+	3 (1)	7
3	9	5

</td>
</tr>
<tr>
<td>Adding multiples of tens using base 10, bead strings, place value counters and arrays. Continue to develop understanding of visually represented tens.

50 = 30 + 20
 

_____ = 10 10 10 + 10 10

  </td>
<td> 
3 tens + 5 tens = _____ tens
30 + 50 = _____
 
36 + 40 = □
 

 
</td>
<td>36 + 40 =

40 + 30 = 70
70 + 6 = 76

Looking at how the numbers can be partitioned to add the multiples of ten then ones.

Also to be counting on in tens.36, 46, 56, 66, 76…</td>
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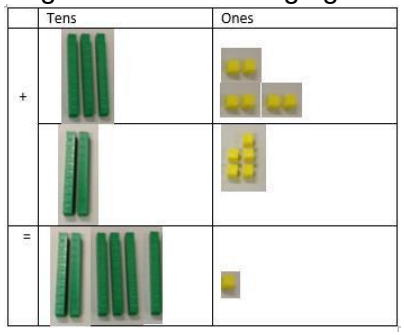
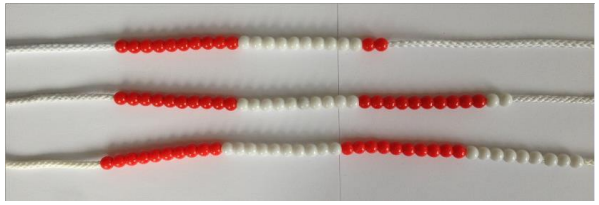
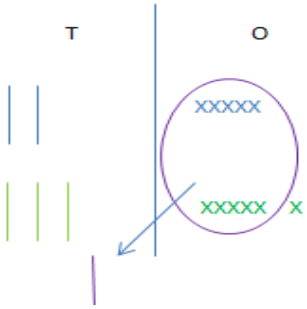
TO + TO using base 10 and bead strings.
Continue to develop understanding of partitioning and place value and use this to support addition. Begin with no exchanging. 36 + 25

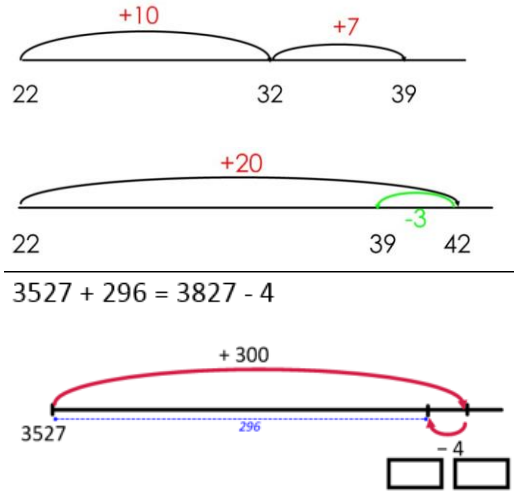


22 + 17 = 39

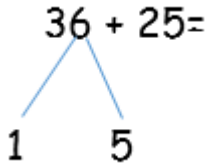


This could be done one of two ways:





Looking for ways to make 10



30 + 20 = 50
5 + 5 = 10
50 + 10 + 1 = 61

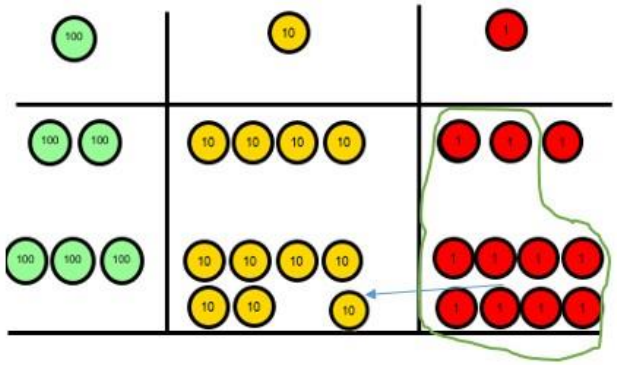
Formal method:

$$\begin{array}{r} 36 \\ +25 \\ \hline 61 \\ \hline {\scriptstyle 1} \end{array}$$

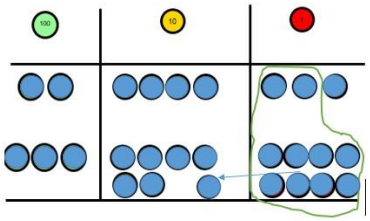
Students can also begin to round to the nearest ten then adjust accordingly.

Round the 17 up to 20, then subtract three.

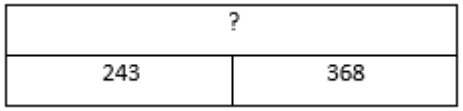
Use of place value counters to add HTO + TO, HTO + HTO etc. once the children have had practice with this, they should be able to apply it to larger numbers and the abstract



Chidren to represent the counters e.g. like the image below

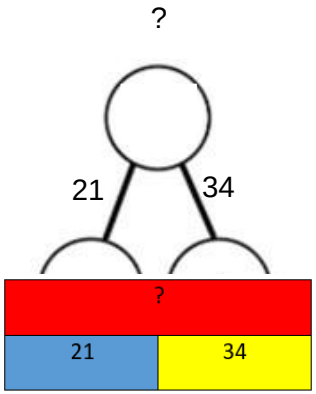


If the children are completing a word problem, draw a bar model to represent what it's asking them to do



$$\begin{array}{r} 243 \\ +368 \\ \hline 611 \\ \hline 1\ 1 \end{array}$$

Fluency variation, different ways to ask children to solve 21+34:



Sam saved £21 one week and £34 another. How much did he save in total?

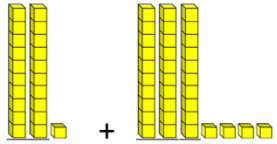
21+34=55. Prove it! (reasoning but the children need to be fluent in representing this)

$$\begin{array}{r} 21 \\ +34 \\ \hline \\ \hline \end{array}$$

21 + 34 =

= 21 + 34

What's the sum of twenty one and thirty four?

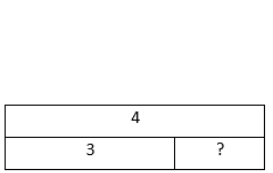
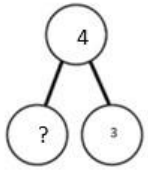
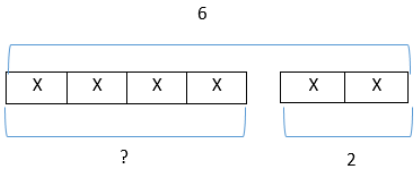
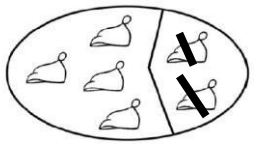
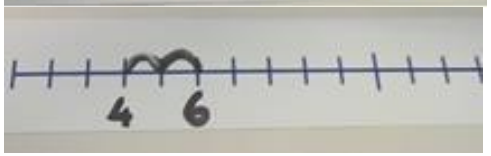
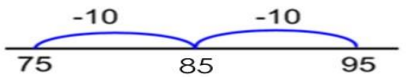
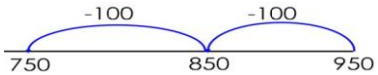


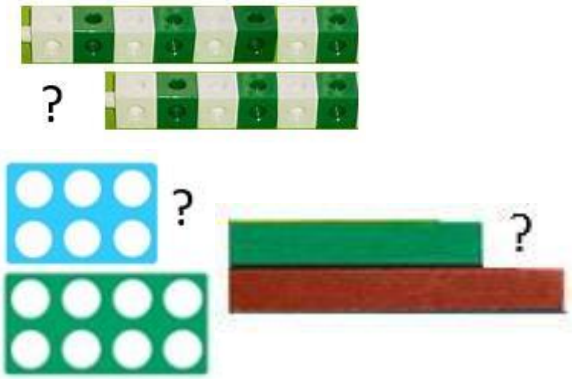
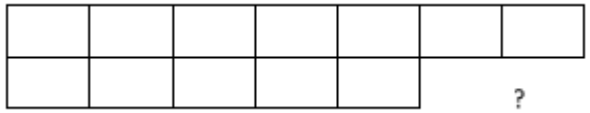
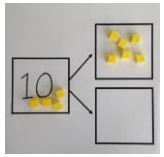
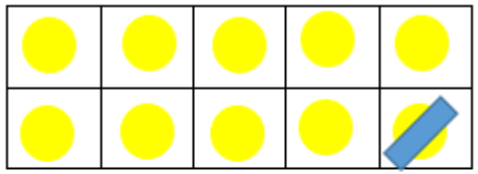
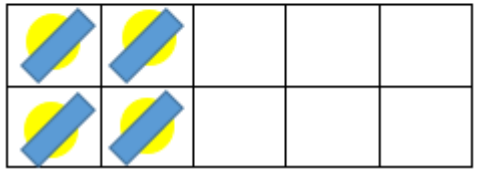
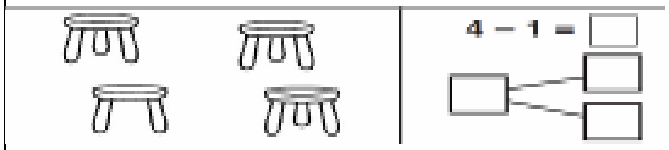
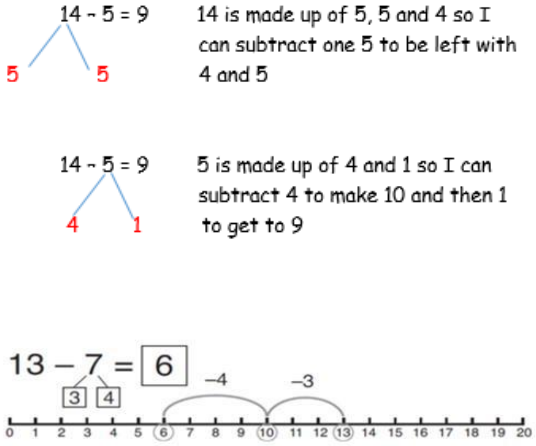
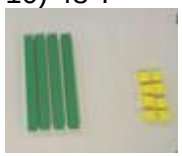
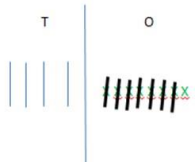
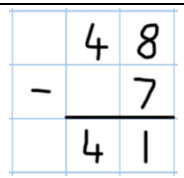
Always use missing digit problems too:

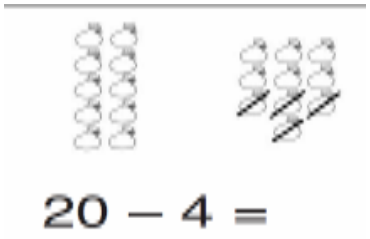
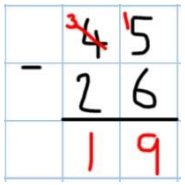
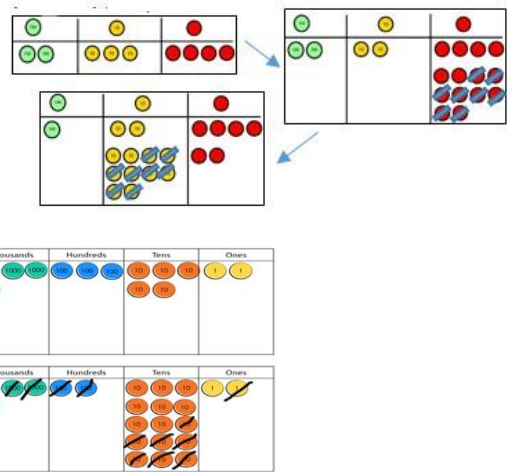
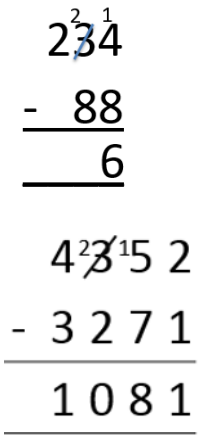
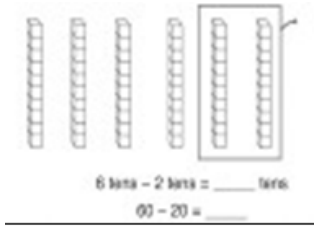
Tens	Ones
	?
?	4

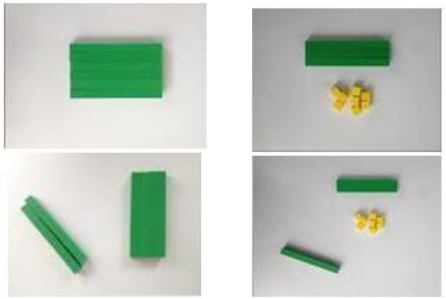
Subtraction-

Key language which should be used: take away, less than, the difference, subtract, minus, fewer, decrease, '7 take away 3, the difference is four'

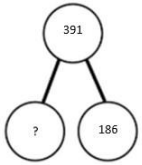
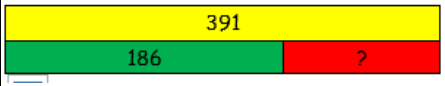
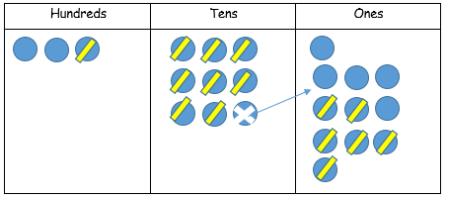
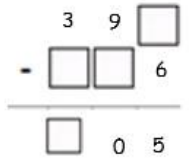
Concrete	Pictorial	Abstract
Physically taking away and removing objects from a whole (use various objects too) rather than crossing out- children will physically remove the objects 4 – 3 = 1 	Children to draw the concrete resources they are using and cross out.  Use of the bar model:  28 - 4 =	**4- 3 =** **= 4 – 3**  
Counting back (using number lines, bead strings or number tracks)   16 – 2 = 14    	Children to represent what they see pictorially e.g.  	    

Finding the difference (using cubes, numicon or Cuisenaire rods, other objects can also be used) 	Children to draw the cubes/other concrete objects which they have used XXXXXXXX XXXXXX Use of the bar model 	Find the difference between 8 and 6. 8 – 6, the difference is ? Children to also explore why 9 - 7 = 8 – 6 (the difference, of each digit, has changed by 1 do the difference is the same- this will help when solving 10000-9987)
Making 10 (using part-part whole, numicon or ten frames) 14 – 5   Children could also do this by subtracting a 5 from the 10. 	Children to present the ten frame pictorially   	14 – 5 = 9 You also want children to see related facts e.g. 15 – 9 = 5 Children to represent how they have solved it e.g. 
Column method (using base 10) 48-7 		48 – 7 = 

Column method (using base 10 and having to exchange) **20-4**  1) Start by partitioning 20 2) Exchange one ten for ten more ones 3) Subtract the ones, then the tens.	**Represent the base 10 pictorially** 	It's crucial that the children understand that when they have exchanged the 10 they still have 45. 45 = 30 + 15 
Column method (using place value counters) 234-88 	Once the children have had practice with the concrete, they should be able to apply it to any subtraction. Like the other pictorial representations, children to represent the counters.	
Subtracting multiples of tens using base 10, bead strings, place value counters and arrays. Continue to develop understanding of visually represented tens		38 - 10 = 30 + 10 = 20 20 + 8 = 28 Looking at how the numbers can be partitioned to subtract the multiples of ten. Counting backwards also in tens. 38, 28…

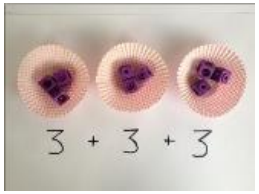
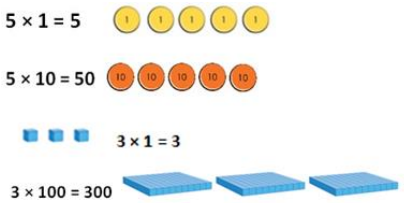
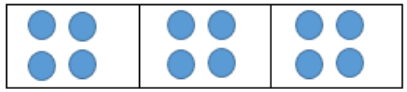
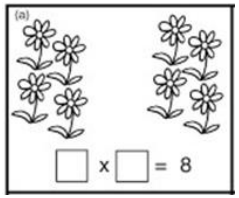
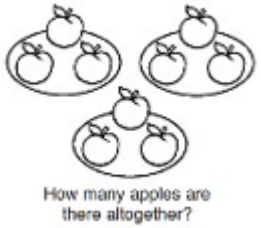
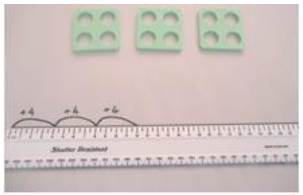
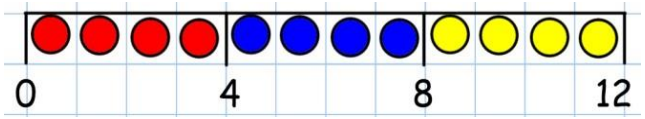
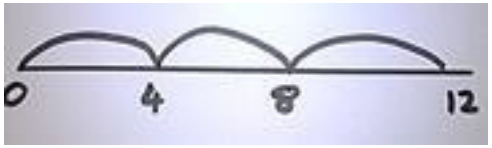
40 = 60 – 20  _____ = 10 10 10 - 10	 38 – 10 = ☐	

Fluency variation, different ways to ask children to solve 391-186:

  	Raj spent £391, Timmy spent £186. How much more did Raj spend? I had 391 metres to run. After 186 I stopped. How many metres do I have left to run?	391 – 186 = 391 – 186 391 -186 ____ Find the difference between 391 and 186 Subtract 186 from 391. What is 186 less than 391?	What's the calculation? What's the answer?  

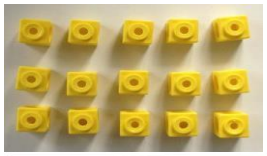
Multiplication-

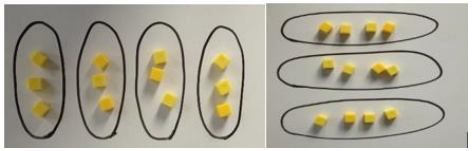
Key language which should be used: double times, multiplied by, the product of, groups of, lots of, 'is equal to' 'is the same as'

Concrete	Pictorial	Abstract
Repeated grouping/repeated addition (does not have to be restricted to cubes) 3 x 4 or 3 lots of 4    3 x 3 = 3 + 3 +3  Students begin to use what they know to derive multiplications.	Children to represent the practical resources in a picture e.g. XX XX XX XX XX XX Use of a bar model for a more structured method   	3 x 4 4 + 4 + 4 2 x 4 = 8 4 + 4 = 8 3 + 3 + 3 = 9 3 x 3 = 9
Use number lines to show repeated groups- 3 x 4  	Represent this pictorially alongside a number line e.g: 	Abstract number line **3 x 4 = 12** 

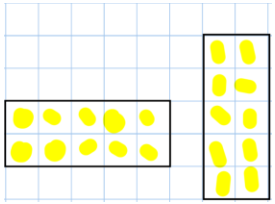
Use arrays to illustrate commutativity (counters and other objects can also be used)

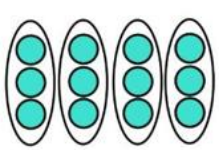
2 x 5 = 5 x 2



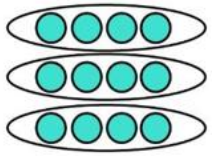


Children to draw the arrays





12 = 3 × 4



12 = 4 × 3

Children to be able to use an array to write a range of calculations e.g.

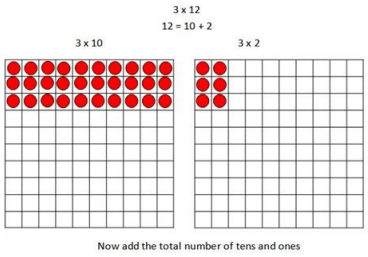
2 x 5 = 10
5 x 2 = 10
2 + 2 + 2 + 2 + 2 = 10
5 + 5 =10

3 x 5 = ☐

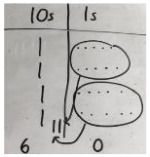
5 x 3 = ☐

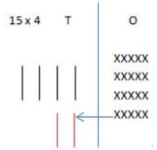
Partition to multiply (use numicon, base 10, Cuisenaire rods)

4 x 15



Children to represent the concrete manipulatives in a picture e.g. base 10 can be represented like:





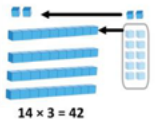
×	10	2
3		

×	10	2
3	30	6

3 x 12 = 36

×	10	4
3	30	12

×	10	4
3	30	12



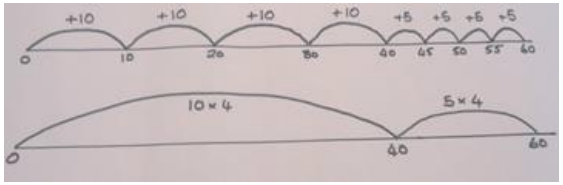
Children to be encouraged to show the steps they have taken

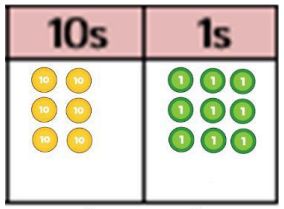
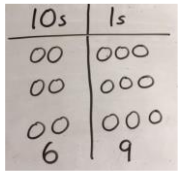
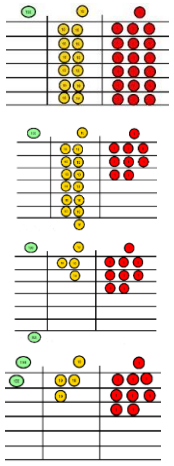
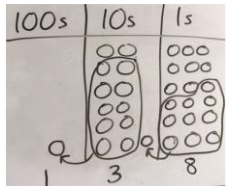
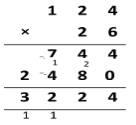
4 x 15

10 5

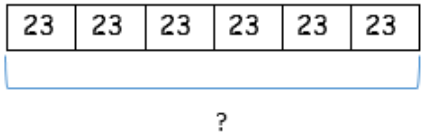
10 x 4 = 40
5 x 4 = 20
40 + 20 = 60

A number line can also be used



Formal column method with place value counters or base 10 (at the first stage- no exchanging) 3 x 23 Make 23, 3 times. See how many ones, then how many tens  6 9	**Children to represent the counters in a pictorial way**   	Children to record what it is they are doing to show understanding 3 x 23 3 x 20 = 60 3 x 3 = 9 20 3 60 + 9= 69 23 × 3 69
Formal column method with place value counters (children need this stage, initially, to understand how the column method works)	Children to represent the counters/base 10, pictorially e.g. the image below.	**6 x 23** 6 x 3 = 18 6 x 20 = 120 120 + 18 = 138
6 x 23  **Step 1**: get 6 lots of 23 **Step 2**: 6 x 3 is 18. Can I make an exchange? Yes! Ten ones for one ten…. **Step 3**: 6 x 2 tens and my extra ten is 13 tens. Can I make an exchange? Yes! Ten tens for one hundred… **Step 4**- what do I have I each column?	  	The aim is to get to the formal method but the children need to understand how it works. 6 x 23 = 23 × 6 138 1 1
When children start to multiply 3d x 3d and 4d x 2d etc, they should be confident with the abstract: To get 744 children have solved 6 x 124 To get 2480 they have solved 20 x 124		

Fluency variation, different ways to ask children to solve 6 x 23:



With the counters, prove that 6 x 23 = 138

Why is 6 x 23 = 32 x 6?

Mai had to swim 23 lengths, 6 times a week. How many lengths did she swim in one week?

Tom saved 23p three days a week. How much did he save in 2 weeks?

Find the product of 6 and 23

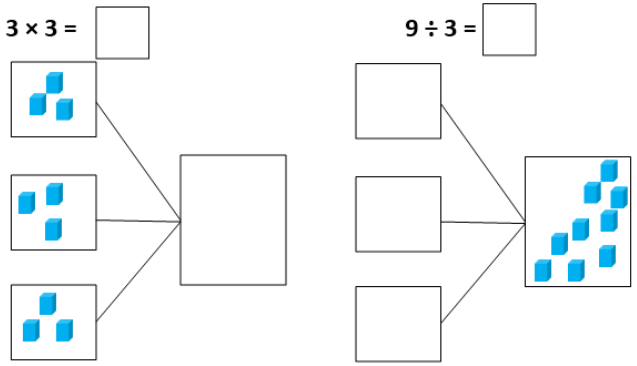
6 x 23 =

☐ = 6 x 23

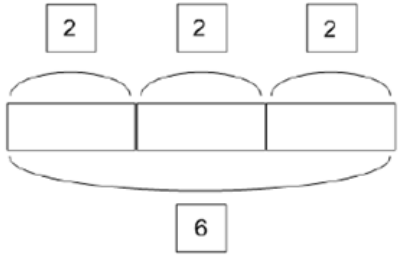
$$\begin{array}{r} 6 \\ \times\ 23 \\ \hline \end{array} \qquad \begin{array}{r} 23 \\ \times\ 6 \\ \hline \end{array}$$

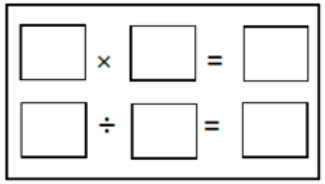
What's the calculation? What's the answer?

There are three equal parts. Each part has a value of three. What is the whole?



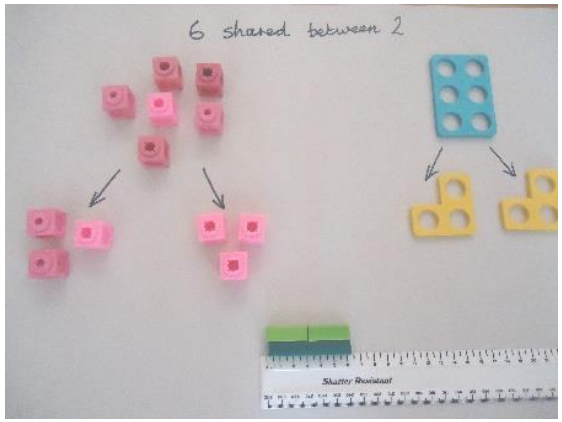
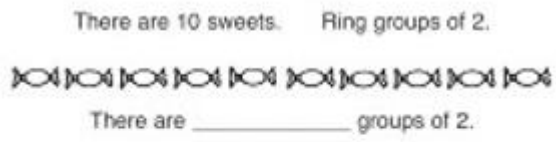
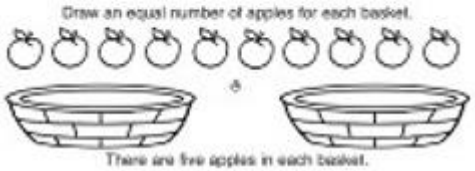
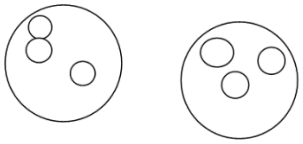
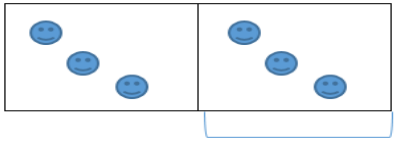
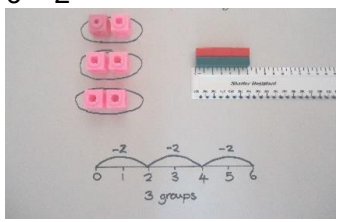
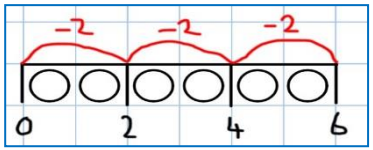
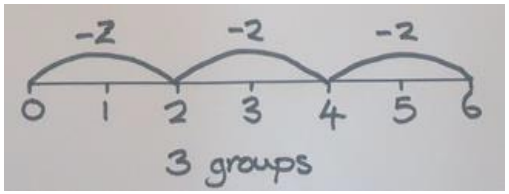
What multiplication and division equations can you write for each bar model? Prove that the equations are correct using……

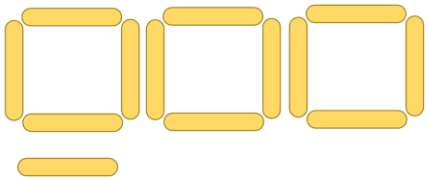
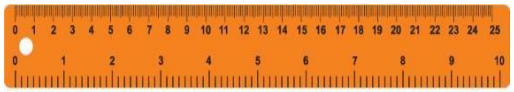
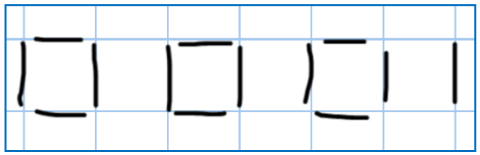
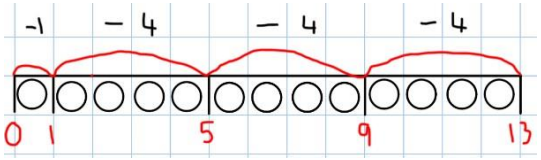
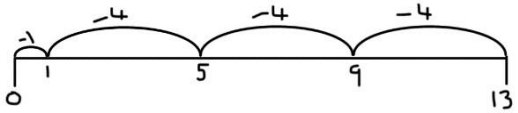
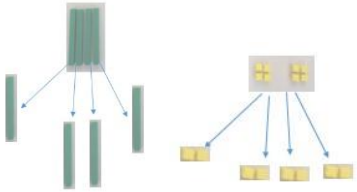
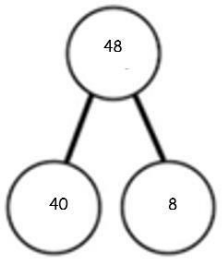
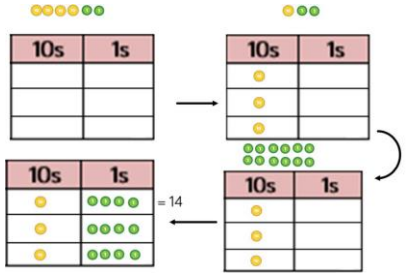
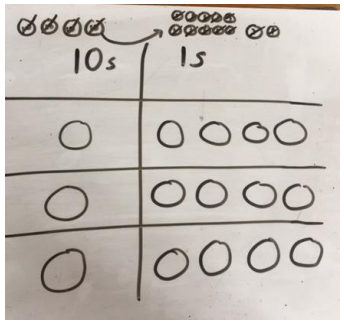


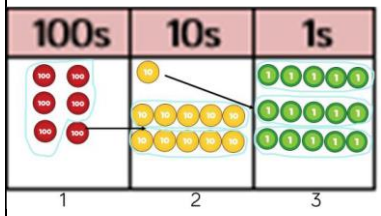
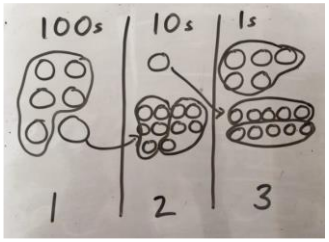


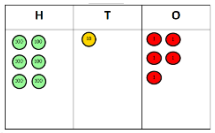
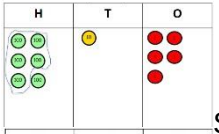
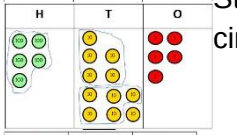
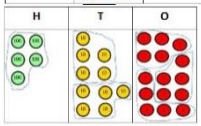
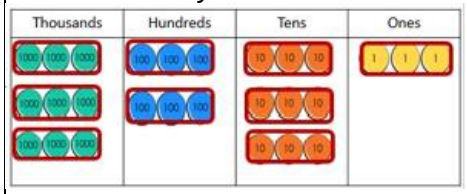
Division-

Key language which should be used: share, group, divide, divided by, half, 'is equal to' 'is the same as'

Concrete	Pictorial	Abstract
6 shared between 2 (other concrete objects can also be used e.g. children and hoops, teddy bears, cakes and plates)   	   This can also be done in a bar so all 4 operations have a similar structure: 	10 ÷ 2 = 5 10 5 | 5 6 ÷ 2 = 3 What's the calculation? 3 | 3
Understand division as repeated grouping and subtracting 6 ÷ 2 		Abstract number line 

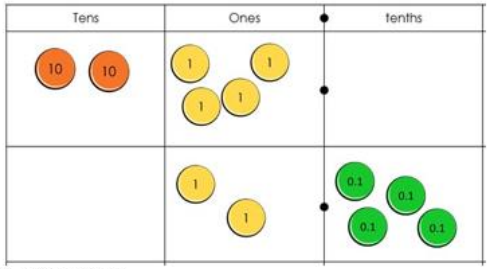
2d ÷ 1d with remainders 13 ÷ 4 – 3 remainder 1	Children to have chance to represent the resources they use in a pictorial way e.g. see below:	13 ÷ 4 – 3 remainder 1 Children to count their times tables facts in their heads
Use of lollipop sticks to form wholes  Use of Cuisenaire rods and rulers (using repeated subtraction) 	 	
2d divided by 1d using base 10 (no remainders) SHARING **48 ÷ 4 = 12**  Start with the tens.	Children to represent the base 10 and sharing pictorially**.**	48 ÷ 4  4 tens ÷ 4 = 1 ten 8 ones ÷ 4 = 2 ones 10 + 2 = 12
Sharing using place value counters. 		42 ÷ 3 = 14 42 = 30 + 12 30 ÷ 3 = 10 12 ÷ 3 = 4 10 + 4 = 14

Short division using place value counters to group. 615 ÷ 5  1. Make 615 with place value counters. 2. How many groups of 5 hundreds can you make with 6 hundred counters? 3. Exchange 1 hundred for 10 tens. 4. How many groups of 5 tens can you make with 11 ten counters? 5. Exchange 1 ten for 10 ones. 6. How many groups of 5 ones can you make with 15 ones?	Represent the place value counters pictorially.  	Children to the calculation using the short division scaffold. $\begin{array}{r} 123 \\ 5\overline{)\,6^{1}1^{1}5} \end{array}$

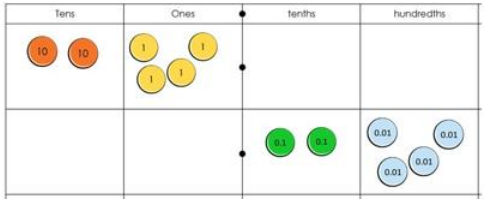
Use of the 'bus stop method' using grouping and counters. Key language for grouping- how many groups of X can we make with X hundreds'- ***this can also be done using sharing!*** 615 ÷ 5 Step 1: make 615  Step 2: Circle your groups of 5  Step 3: Exchange 1H for 10T and circle groups of 5  Step 4: exchange 1T for 10 ones and circles groups of 5  3231 divided by 3 	This can easily be represented pictorially, till the children no longer to do it. It can also be done to decimal places if you have a remainder!	$$\begin{array}{r} 123 \\ 5\overline{)615} \\ {\scriptstyle 1\,1} \end{array}$$ $$\begin{array}{r} 3231 \\ 3\overline{)9693} \end{array}$$

Division of a one or a two-digit number by 10 and 100, identifying the value of the digits in the answer.

24 ÷ 10 = 2.4

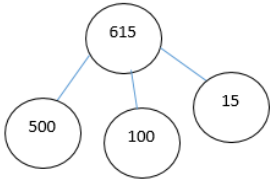


24 ÷ 100 = 0.24



Fluency variation, different ways to ask children to solve 615 ÷ 5:

Using the part whole model below, how can you divide 615 by 5 without using the 'bus stop' method?



I have £615 and share it equally between 5 bank accounts. How much will be in each account?

615 pupils need to be put into 5 groups. How many will be in each group?

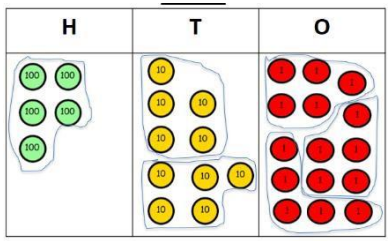
$5\overline{)615}$

615 ÷ 5 =

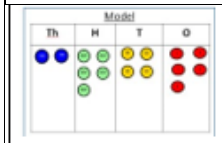
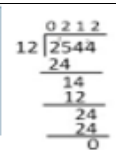
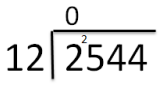
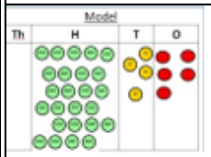
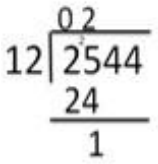
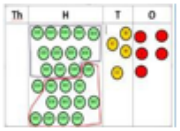
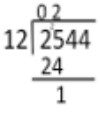
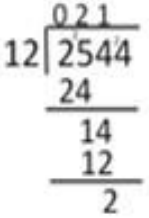
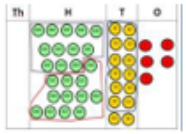
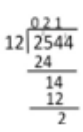
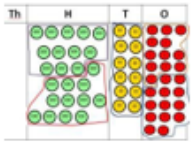
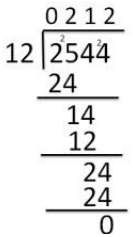
☐ = 615 ÷ 5

How many 5's go into 615?

What's the calculation? What's the answer?



Long division

Concrete	Pictorial	Abstract
  2544 ÷ 12 How many groups of 12 thousands. do we have? None	Children to represent the counters, pictorially and record the subtractions beneath.	 Step one- exchange 2 thousand for 20 hundreds so we now have 25 hundreds.
 Exchange 2 thousand for 20 hundreds.		 Step two- How many groups of 12 can I make with 25 hundreds? The 24 shows the hundreds we have grouped. The one is how many hundreds we have left.
  How many groups of 12 are in 25 hundreds? 2 groups. Circle them. We have grouped 24 hundreds so can take them off and we are left with one.		 Exchange the one hundred for 10 tens. How many groups of 12 can I make with 14 tens? The 14 shows how many tens I have, the 12 is how many I grouped and the 2 is how many tens I have left.
  Exchange the one hundred for ten tens so now we have 14 tens. How many groups of 12 are in 14? 1 remainder 2.		
 Exchange the two tens for twenty ones so now we have 24 ones. How many groups of 12 are in 24? 2		 Exchange the 2 tens for 20 ones. The 24 is how many ones I have grouped and the 0 is what I have left.